BSE 5663 - Analysis of Frequency Data
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2x2 contingency tables
Sensitivity and Specificity

2x2 table relating test results to an outcome or disease state

		True state, sometimes validated by "gold standard"		
		Outcome present (+)	Outcome absent (-)	
Test Result	positive test (+)	TP	FP	total who test positive
	negative test (-)	FN	TN	total who test negative
		total with condition	total without condition	total

Prevalence = (TP+FN)/ total, the proportion who truly have the condition.

Important conditional probabilities:

Sensitivity and specificity. We assume that these are stable properties of a test and are unrelated to disease prevalence. We should examine this assumption more carefully for many tests.

Sensitivity = TP/(TP+FN)
the probability that someone with the condition will test positive.

Specificity = TN/(TN+FP)
the probability that someone without the condition will test negative.

Predictive Values. These depend on the prevalence of the condition in the population to which the test is applied.

Predictive Value Positive (PV+) = TP/(TP+FP)
the probability that someone with a positive test truly has the condition.

Predictive Value Negative (PV-) = TN/(TN+FN)
the probability that someone with a negative test truly does not have the condition.

Spin and Snout: clinical decision-making mnemonics

SPIN: use a SPecific test to rule IN or confirm a clinical hypothesis. Because highly specific tests generate very few false positives, a positive test is likely to be a true positive.

SNOUT: use a SeNsitive test to rule OUT or discard a clinical hypothesis. Because highly sensitive tests generate very few false negatives, a negative test is likely to be a true negative.

Likelihood Ratios

Let D represent disease,
so that D+ represents the true presence of disease and D- represents the true absence of disease.

Let T represent the clinical test,
so that T+ represents a positive test and T- represents a negative test.

Sensitivity (Sn) = p(T+|D+)
1-Sensitivity (1-Sn) = p(T-|D+)

Specificity (Sp) = p(T-|D-)
1-specificity (1-Sp) = p(T+|D-)

Therefore, Sn/(1-Sp) = p(T+|D+) / p(T+|D-)
This is LR+, the likelihood of a positive test in a patient with the disease compared to the likelihood a positive test in a patient without the disease.

If we multiply a test's likelihood ratio with the pretest odds that one has a disease (a quantity for which clinical knowledge can generate an informed estimate or guess), the product is the *post-test odds* the person has the disease!!!!!

For example, if before performing the test, we believe the odds a person has a disease is 1 (even odds; he or she is as likely to have the disease as not), and then we obtain a positive result on a clinical test whose Sn and Sp equate to a likelihood ratio of 4, then the post-test odds of disease are 4. Odds of 4 equate to a probability of 4/(1+4) or 0.8. The positive test heightens our suspicion of the disease's actual presence.

How does this work? The clinician estimates the pre-test probability of disease to be p(D+). The equivalent pretest odds is then p(D+)/ p(D-).

The product of the likelihood and the pretest odds is:

$$\frac{p(T+|D+)}{p(T+|D-)} * \frac{p(D+)}{p(D-)} = \frac{p(T+\cap D+)}{p(T+\cap D-)}$$

Dividing the expression's numerator and denominator by the probability of a positive test, p(T+)

$$\frac{p(T+\cap D+)\ /\ p(T+)}{p(T+\cap D-)\ /\ p(T+)} = \frac{p(D+|T+)}{p(D-|T+)}$$

which expresses the post-test odds of disease.

An analogous relationship exists if we wish to focus on the odds that one does not have the disease. This is perhaps less often calculated, and requires a different likelihood:

Sp/(1-Sn) = p(T-|D-) / p(T-|D+)

This is the LR-, the likelihood of obtaining a negative test if disease is truly absent compared to the likelihood of obtaining a negative test if the disease is truly present.

References

Sackett, D.L., Haynes, R. B., Guyatt, G.H., & Tugwell, P. (1991). *Clinical epidemiology: A basic science for clinical medicine*. (2nd ed.). Boston: Little, Brown.

Simon, S. (July 8, 2008). *Mathematical derivation of the odds form of Bayes theorem*. Retrieved January 29, 2010, from http://www.childrens-mercy.org/stats/weblog2006/OddsFormBayes.asp